Essence of Geometric Algebra

Kenichi Kanatani

Professor Emeritus, Okayama University, Japan

Introduction

We introduce "geometric algebra", which recently has been attracting attention of many computer vision and graphics researchers.

Many books and articles on geometric algebra start with definitions of symbols and terminologies followed by identities and relationships among them. This often makes beginners shy away. This tutorial takes an alternative approach: the emphasis is on the background mathematics, including the Hamilton algebra, the Grassmann algebra, and the Clifford algebra. In the end, it is shown how these are combined as geometric algebra.

In order to illustrate the close connection to computer vision applications, we also describe imaging geometry of fisheye lens and omnidirectional cameras using parabolic, hyperbolic, and elliptic mirrors, which play more and more important roles in computer vision and robotics applications as their price goes down.

References: For Japanese participants, a recent book in Japanese "Geometry and Algebra: Hamilton, Grasmann, and Clifford" (Morikita, 2014) is available as a textbook, on which this tutorial is based. It is not necessary to read this book for hearing this tutorial, although it would certainly help understand the details. For English-speaking audience, the following book is recommended:

> L. Dorst, D. Fortijne, and S. Mann, *Geometric Algebra for Computer Science: An Object-Oriented Approach to Geometry*, Morgan Kaufmann, Burlington, MA, U.S.A., 2007.

Course Description

1. **Introduction:** history, algebras, formal sums
2. **Hamilton's quaternion algebra:** quaternions, conjugates, inverse, rotation
3. **Grassmann's outer product algebra:** outer product, bivectors, trivectors, multivectors, vector calculus, vector product, scalar triple product
4. **Clifford's geometric algebra:** geometric product, multivectors, parities, Hamilton algebra, Grassmann algebra, inverse, rotation
5. **Grassmann-Cayley algebra:** 4-D homogeneous space, lines, planes, Plücker coordinates projective space, duals, joins, meets, duality theorems
6. **Conformal geometric algebra:** 5-D non-Euclidean conformal space outer product, direct representation, dual representation, lines, planes, circles, spheres versors, conformal mapping, reflector, invertor, translator, rotor, dilator, motor

7. **Camera imaging geometry:** perspective cameras, image sphere, stereographic projection, inversion, fisheye lens, omnidirectional cameras, parabolic mirrors, hyperbolic mirrors, elliptic mirrors, vanishing points, focal length computation

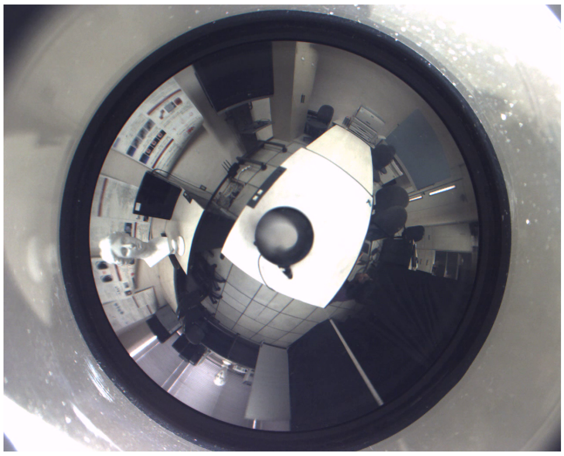
8. **Conclusions:** significance, application potential, current research trend.

Anticipated Audience

This tutorial is intended for students of graduate levels and general researchers of computer vision. People coming merely out of mathematical curiosity about geometry and algebra are very welcome, but those who want to use geometric algebra in real situations can learn many things.

No particular background knowledge is necessary other than the standard mathematics taught to all students of science and engineering.

The description of the imaging geometry of fisheye lens and omnidirectional cameras should attract attention of interested researchers, since they play more and more important role in computer vision and robotics applications as their price goes down.



Biography of the Tutor

Kenichi Kanatani, Professor Emeritus, IEEE Fellow, received his B.E., M.S., and Ph.D. in applied mathematics from the University of Tokyo in 1972, 1974 and 1979, respectively. After serving as Professor of computer science at Gunma University, Japan, till March 2001, he moved to Okayama University, where he was Professor of computer science. He retired in March 2013. He is the author of “Group-Theoretical Methods in Image Understanding” (Springer 1990), “Geometric Computation for Machine Vision” (Oxford University Press, 1993) and “Statistical Optimization for Geometric Computation: Theory and Practice” (Elsevier 1996; reprinted Dover 2005).