Kanatani, Kenichi * **1319.68004**
Understanding geometric algebra. Hamilton, Grassmann, and Clifford for computer vision and graphics.
Boca Raton, FL: CRC Press (ISBN 978-1-4822-5950-6/hbk). xv, 192 p. £ 49.99 (2015).
The present book is a monograph on geometric algebra. Departing from the approach of describing and computing geometry by means of vector and tensor calculus and matrix computation based on linear algebra, in the late 20th century and early 21st century, scientists and engineers revisited 19th-century developments like Hamilton algebra, Grassmann algebra, and Clifford algebra and developed a new formulation called "geometric algebra". This made a significant impact on scientific research and engineering applications in areas such as robotic arm control, computer graphics, and computer vision. In comparison to some existing textbooks on geometric algebra, this book begins with separate discussions of various algebras that constitute the background and then shows how they are finally combined to define geometric algebra. Following the fundamental principle of algebra of defining operations on symbols, a vector in this book is viewed as a symbol to represent a geometric object, for describing 3D Euclidean geometry. All algebras in this book are defined with respect to an orthogonal coordinate system, but a quantity called the metric tensor is introduced to cover the case of non-orthogonal coordinate systems. Hamilton's quaternion algebra and Grassmann's outer product algebras are described in Chapters 4 and 5 respectively. Besides the operations of inner product of vectors and outer product of the Graussmann algebra, a new operation called geometric product (or Clifford product) is introduced and it is shown that the inner and outer products can be defined in terms of the geometric product. Chapter 7 describes points and lines in 3D as objects in 4D, by introducing a fourth symbol to the three basis elements. Conformal geometry, which is the main ingredient of geometric algebra, is described in Chapter 8, where spheres and circles along with their conformal transformations are considered. Introducing an additional new symbol to the 4D space defined in Chapter 7, it is observed that the resulting 5D space is non-Euclidean. Thus, 3D Euclidean geometry is realized in the 5D non-Euclidean space. Camera imaging geometry that involves inversion is considered in the last Chapter 9. Thus, starting with conventional perspective projection cameras (see [*O. Faugeras* and *Q.-T. Luong*, The geometry of multiple images. The laws that govern the formation of multiple images of a scene and some of their applications. Cambridge, MA: MIT Press (2001; Zbl 1002.68183)]), fisheye lens cameras (see [*K. Kanatani*, "Calibration of ultrawide fisheye lens cameras by eigenvalue minimization", IEEE Trans. Pattern Anal. Mach. Intell. 35, No. 4, 813–822 (2013; `doi:10.1109/TPAMI.2012.146`)]) are described. Further, imaging geometry of omnidirectional cameras (see [*R. Benosman* (ed.) and *S. B. Kang* (ed.), Panoramic vision. Sensors, theory, and applications. New York, NY: Springer (2001; Zbl 1027.68132)] are analyzed in this chapter. Several software tools are available for executing geometric algebra, but the purpose of the book is to bring about a deeper insight and interest in the theory on which these tools are based. *H. P. Dikshit (Bhopal)*