Comments on "Nonparametric Segmentation of Curves Into Various Representations"

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Abstract—I point out the existence of a theoretical difficulty that underlies the curve segmentation problem studied by Rosin and West and present a possible solution to it.

Index Terms—Curve segmentation, line fitting, conic fitting, pattern recognition, geometric AIC.

1 Introduction

ROSIN and West [8] presented an algorithm (let me call it the *RW algorithm*) for automatically transforming a curve into straight-line segments and higher-order features, such as circular arcs, conic segments, and splines. Here, I point out the existence of a theoretical difficulty that underlies this problem and present a possible solution to it. This does not mean that there is something wrong with the RW algorithm; the difficulty is of a purely theoretical nature, and the RW algorithm avoids it by heuristics.

2 Segmentation Into Features of Different Types

The problem is that segmenting a curve into features of the same type is one thing, and segmenting it into features of *different types* is quite another. This is because different features have *inclusion relations* among themselves: A line segment is a kind of circular arc (with an infinite radius), which is a kind of conic segment, which is a kind of higher order feature, and so on. The goodness of a feature is evaluated by some distance measure between the selected feature and the data. The RW algorithm adopted the maximum deviation (l_∞-distance) divided by the length of the feature, which Rosin and West called the *significance measure*, but other measures, such as the average absolute deviation (l_1-distance) and the square average deviation (l_2-deviation), could also serve the same purpose.

The RW algorithm first transforms a curve into line segments. Suppose we want to check if they can be replaced by a single feature. If they are replaced by a single line segment that connects the endpoints, the distance measure is uniquely determined. If they are to be replaced by a circular arc that connects the endpoints, one degree of freedom remains, so the one with the smallest distance measure is chosen. If a conic segment that connects the endpoints is to be fitted, there remain three degrees of freedom, with respect to which the distance measure is minimized. Hence, a higher-order feature with a larger degree of freedom always has a smaller distance measure. It follows that any algorithm for segmenting a curve into features of different types is faced with the problem of how to *suppress* higher order features, or how *not* to adopt the "optimal" solution.

The strategy of the RW algorithm is to (arbitrarily) fix the order of construction, search, and merge. For example, consider the first stage of polygonal approximation. If we want to divide a curve

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into N segments with the endpoints fixed, the $N - 1$ break points can theoretically be anywhere on the curve, and we can minimize, say, the sum of the distance measures of the resulting N segments by adjusting the $N - 1$ degrees of freedom. Evidently, this process would be costly, so the RW algorithm takes the recursive subdivision approach: The curve is divided into two segments by choosing the point of maximum deviation as the break point (there is no guarantee that the significance measures of the resulting segments are lowered), and this process is recursively applied to the resulting segments, thereby avoiding an exhaustive search.

3 Class Selection by Geometric AIC

Let me state the problem in general terms. The task is to find an optimal class for a given input pattern. It appears that we need to choose only the one that is the "closest" to the input by introducing some distance measure. This is the basic discipline of *pattern recognition*. However, it is tacitly assumed in pattern recognition that the classes into which the input is to be classified are *disjoint*. If class A is *included* in class B, class A is never chosen, because the distance to class A is always not smaller than that to class B (Fig. 1). Thus, higher-order features are always favored over lower-order ones (the RW algorithm does not exactly fit in this framework, though).

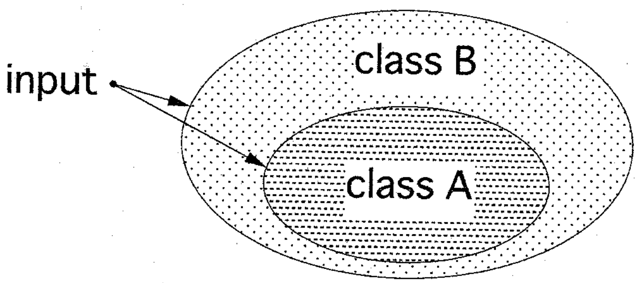

Fig. 1. Class A is never chosen, whatever distance measure is used.

In order to give every class a fair chance, we must introduce another criterion that favors classes that are included in others. A similar problem appears in statistics, and the *AIC* (*Akaike Information Criterion*) gives one solution [1]. However, the AIC is formulated for a *statistical inference* formalized as estimating the parameter of a statistical distribution from multiple samples chosen from it. As a result, the AIC cannot be applied to *geometric inference* in its original form in general; it can be applied to curve and surface segmentation only when the problem has a special form called (linear/nonlinear) *regression* [2]. However, if we go back to the principle that gives rise to the AIC, we can obtain a similar criterion, called the *geometric AIC* [4], applicable to geometric inference in general.

The principle underlying the geometric AIC is the *robustness to perturbation*. As an illustration, consider curve approximation. Suppose curve l is approximated by the best feature a chosen from class A. Consider a hypothetical curve l' obtained by perturbing the curve l. Suppose l' is approximated by the best feature a' also chosen from class A. Intuitively, feature a is "robust" when *if a and a' are close, then a is also a good approximation to l'*. It follows that if class A is included in class B, a fit from class A is always more robust than one from class B, because the set A' of features in class A that are within a distance from the best feature in A is smaller than the set B' of features in class B that are within the same distance from the best feature in B. In other words, the set of curves that are approximated by the features in B' are larger than the set of curves that are approximated by the features in A'.

Skipping the derivation (see [4] for the details), it is concluded as follows. Suppose a curve is divided into N segments, and we want to find the best feature that passes through the endpoints. Let us adopt the sum of the squared distances of the $N - 1$ break points from the fitted feature as the distance measure. Let $a \in A$ be the best feature in class A, and let $b \in B$ be the best feature in class B. Let D_a and D_b be their distance measures. Suppose class A is included in class B: $A \subset B$. Let f_A and f_B be the degrees of freedom of classes A and B, respectively, that remain under the constraint that features should pass through the endpoints. According to the geometric AIC, class A is preferred to class B if

$$\frac{D_a}{D_b} < 1 + \frac{2(f_b - f_a)}{N - f_b - 1}. \tag{1}$$

For example, a line segment is preferred to a circular arc for $N > 2$ if

$$\frac{D_{\text{line}}}{D_{\text{circle}}} < 1 + \frac{2}{N - 2}, \tag{2}$$

and a circular arc is preferred to a conic segment for $N > 4$ if

$$\frac{D_{\text{circle}}}{D_{\text{conic}}} < 1 + \frac{4}{N - 4}. \tag{3}$$

In order to apply such criteria, we need to compute an optimal fit that minimizes the sum of the squared distances from the break points and its minimum value efficiently. For line fitting, this is easily done by the usual least-squares method or its modification [6]. For conic fitting, this is achieved by a technique called *renormalization* [7], a prototype of which was given by Kanatani [3].

4 Concluding Remarks

The kind of reasoning stated above is expected to play a crucial role for not only curve segmentation but also many other problems where model selection is involved [5], [9], [10].

References

[1] H. Akaike, "A New Look at the Statistical Model Identification," *IEEE Trans. Automation Control*, vol. 19, no. 6, pp. 716–723, 1974.

[2] K.L. Boyer, M.J. Mirza, and G. Ganguly, "The Robust Sequential Estimator: A General Approach and Its Application to Surface Organization in Range Data," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 16, no. 10, pp. 987–1,001, Oct. 1994.

[3] K. Kanatani, "Statistical Bias of Conic Fitting and Renormalization," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 16, no. 3, pp. 320–326, Mar. 1994.

[4] K. Kanatani, *Statistical Optimization for Geometric Computation: Theory and Practice*. Amsterdam: Elsevier Science, 1996.

[5] K. Kanatani, "Comments on 'Symmetry as a Continuous Feature,'" *IEEE Trans. Pattern Analysis and Machine Intelligence*, in press.

[6] Y. Kanazawa and K. Kanatani, "Optimal Line Fitting and Reliability Evaluation," *IEICE Trans. Information and Systems*, vol. E79-D, no. 9, pp. 1,317-1,322, Sept. 1996.

[7] Y. Kanazawa and K. Kanatani, "Optimal Conic Fitting and Reliability Evaluation," *IEICE Trans. Information and Systems*, vol. E79-D, no. 9, pp. 1,323-1,328, Sept. 1996.

[8] P.L. Rosin and G.A.W. West, "Nonparametric Segmentation of Curves Into Various Representations," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 17, no. 12, pp. 1,140–1,153, Dec. 1995.

[9] I. Triono and K. Kanatani, "Automatic Recognition of Regular Figures by Geometric AIC," *Proc. IAPR Workshop Machine Vision Applications*, pp. 393-396, Tokyo, Nov. 1996.

[10] R. Zabrodsky, S. Peleg, and D. Avnir, "Symmetry as a Continuous Feature," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 17, no. 12, pp. 1,154–1,166, Dec. 1995.