Optimal Geometric Computation and Model Selection*

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1. Introduction

Here, we focus on optimal computation of the *fundamental matrix*, the *flow fundamental matrix*, and the *homography* in the presence of image noise. The C++ source code of our programs and related articles are available at our Web site[1]. The underlying theory is described in [4, 5].

Model of image noise

In order to compute geometric quantities in the presence of noise, numerous heuristic criteria have been proposed one after another—least squares, algebraic distance minimization, weighted least squares, and their hybrid to name a few. However, the first step is to study the behavior of the noise. Once we establish a mathematical model of noise, we can obtain an optimal solution *under that model* by doing statistical analysis.

Outliers

Because "outliers" caused by mismatches of corresponding points are difficult to be modeled statistically, statistical modeling of noise itself has been avoided. However, one should apply a statistically optimal algorithm once one has removed outliers by an algorithm that is effective in detecting them.

Theoretical accuracy bound

It has been customary to compare a proposed method with "previously proposed methods" by simulations and real-image experiments. However, optimality of a method is established only by comparing it with a *theoretical accuracy bound*.

Reliability evaluation

If an algorithm attains the theoretical accuracy bound, that algorithm is strictly optimal; we have a guarantee that no further improvement is possible. Moreover, we can evaluate the reliability of the computed solution by computing that bound.

Model selection

The estimation computation often becomes unstable if a large number of unknowns are involved. If the solution is constrained to be in a small subspace of the parameter space, we can stably search that subspace or the *model*. However, we do not know which model is correct a priori. So, we need an appropriate model selection criterion such as the *geometric AIC*.

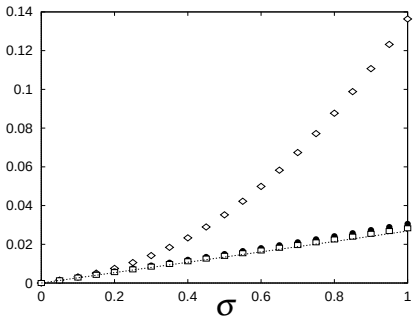

Figure 1: Accuracy of computing the fundamental matrix.

2. Fundamental Matrix

Given two sets of points $\{(x_\alpha, y_\alpha)\}$ and $\{(x'_\alpha, y'_\alpha)\}$, we want to compute a matrix $\boldsymbol{F}$ of rank 2 that would satisfy the following *epipolar equation* in the absence of noise:

$$\begin{pmatrix} x_\alpha & y_\alpha & 1 \end{pmatrix} \boldsymbol{F} \begin{pmatrix} x'_\alpha \\ y'_\alpha \\ 1 \end{pmatrix} = 0 \qquad (1)$$

The matrix $\boldsymbol{F}$ is called the *fundamental matrix*.

Fig. 1 plots the root-mean-squares error of the fundamental matrix $\boldsymbol{F}$ (normalized to $\|\boldsymbol{F}\| = 1$) computed in the presence of random Gaussian noise of standard deviation σ (pixels) over 100 trials. The symbol ● is for the solutions computed by a technique called *renormalization*, and the symbol □ is for the solutions obtained by correcting the renormalization solutions so that the constraint $\det \boldsymbol{F} = 0$ is statistically optimally satisfied. We see that the theoretical accuracy bound, indicated by the dotted line in the figure, is already attained. The symbol ◇ is for the solutions computed by the least-squares method (*algebraic distance minimization*) [2, 3].

3. Flow Fundamental Matrix

Given a set of points $\{(x_\alpha, y_\alpha)\}$ and the velocities (*optical flow*) $\{(\dot{x}_\alpha, \dot{y}_\alpha)\}$ defined there, we want to compute an antisymmetric matrix $\boldsymbol{W}$ and a symmetric matrix $\boldsymbol{C}$ that would satisfy the following *optical flow epipolar equation* in the absence of noise [1]:

$$\begin{pmatrix} x_\alpha & y_\alpha & 1 \end{pmatrix} \boldsymbol{W} \begin{pmatrix} \dot{x}_\alpha \\ \dot{y}_\alpha \\ 0 \end{pmatrix}$$

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[1] `http://www.ail.cs.gunma-u.ac.jp/~kanatani/e`

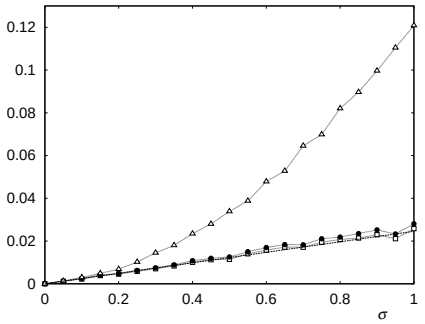


Figure 2: Accuracy of computing the flow fundamental matrix.

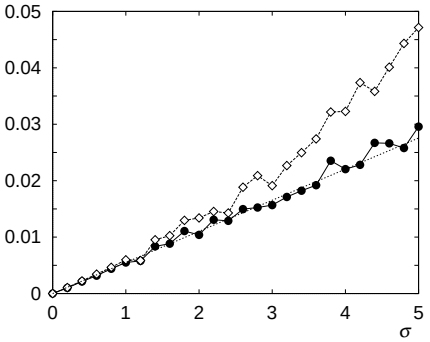


Figure 3: Accuracy of computing the homography matrix.

$$+ \begin{pmatrix} x_\alpha & y_\alpha & 1 \end{pmatrix} \boldsymbol{C} \begin{pmatrix} x_\alpha \\ y_\alpha \\ 1 \end{pmatrix} = 0 \qquad (2)$$

We call $\boldsymbol{F} = \boldsymbol{C} + \boldsymbol{W}$ the *flow fundamental matrix*.

Fig. 2 plots the root-mean-squares error of the flow fundamental matrix $\boldsymbol{F}$ (normalized to $\|\boldsymbol{F}\| = 1$) computed in the presence of random Gaussian noise of standard deviation σ (pixels) over 100 trials. The symbol ● is for the solutions computed by renormalization, and the symbol □ is for the solutions obtained by correcting the renormalization solutions so that what is known as the *decomposability condition* is optimally satisfied. We see that the theoretical accuracy bound, indicated by the dotted line in the figure, is already attained. The symbol △ is for the solutions computed by the least-squares method (algebraic distance minimization) [1].

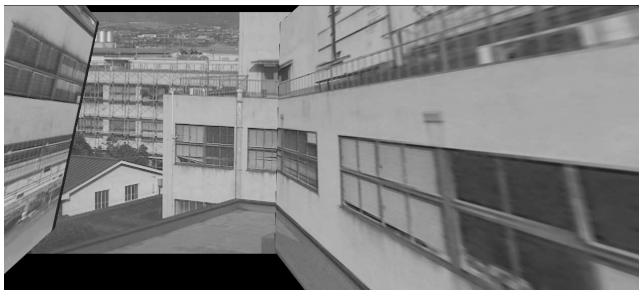
4. Homography

Given two sets of points $\{(x_\alpha, y_\alpha)\}$ and $\{(x'_\alpha, y'_\alpha)\}$, we want to compute a nonsingular matrix $\boldsymbol{H}$ that would define a *homography* in the absence of noise in the form

$$\begin{pmatrix} x'_\alpha \\ y'_\alpha \\ 1 \end{pmatrix} \simeq \boldsymbol{H} \begin{pmatrix} x_\alpha \\ y_\alpha \\ 1 \end{pmatrix}, \qquad (3)$$

where $\simeq$ means that one side is a multiple of the other side by a nonzero constant.

Fig. 3 plots the root-mean-squares error of the homography matrix $\boldsymbol{H}$ (normalized to $\|\boldsymbol{H}\| = 1$) computed in the presence of random Gaussian noise of standard deviation σ (pixels) over 50 trials. The symbol ● is for the solutions computed by renormalization. We see that the theoretical accuracy bound, indicated by the dotted line in the figure, is already attained. The symbol ◇ is for the least-squares method (algebraic-distance minimization).



(a)

(b)

Figure 4: (a) Mosaicing with optimal homography. (b) Mosaicing with model selection.

5. Model Selection

Integrating multiple views of an outdoor scene into a large single image by applying homographies is known as *mosaicing*. If two images overlap along a thin strip, the computation of the homography between them is very unstable. Since actual transformations are usually in a small subgroup of the group of homographies, we can stabilize the computation if we infer an appropriate subgroup, such as affine transformations, similarity transformations, and rigid motions, by using the *geometric AIC* (Fig. 4).

5. Concluding Remarks

Optimization and model selection techniques described here are expected to play an important role in a wide variety of robotics, computer vision, and media processing applications.

References

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