## High Accuracy Computation of Rank-constrained Fundamental Matrix by Efficient Search

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**Abstract** A new method is presented for computing the fundamental matrix from point correspondences over two images: its singular value decomposition (SVD) is optimized by the Levenberg-Marquard (LM) method. There is no need for tentative 3-D reconstruction. The accuracy of the solution is compared with the theoretical bound (the KCR lower bound).

## 1. Proposed Method

The fundamental matrix  $\boldsymbol{F}$  has nine elements, on which the normalization  $\|\boldsymbol{F}\| = 1$  and the rank constraint det  $\boldsymbol{F} =$ 0 are imposed. So, it has seven degrees of freedom. In this paper, we adopt the 7-degree parameterization of Bartoli and Sturm [1]: we express  $\boldsymbol{F}$  by its singular value decomposition (SVD)  $\boldsymbol{F} = \boldsymbol{U} \text{diag}(\sigma_1, \sigma_2, 0) \boldsymbol{V}^{\top}$ , where  $\boldsymbol{U}$  and  $\boldsymbol{V}$  are orthogonal matrices, and  $\sigma_1$  and  $\sigma_2$  are the singular values. Since the normalization  $\|\boldsymbol{F}\| = 1$  is equivalent to  $\sigma_1^2 + \sigma_2^2 = 1$ , we introduce the parameterization  $\sigma_1 = \cos \theta$ ,  $\sigma_2 = \sin \theta$ .

Using this parameterization, we minimize the maximum likelihood (ML) cost function by the Levenberg-Marquard (LM) method. Following Bartoli and Sturm [1], we use the "Lie algebraic method" for optimizing the orthogonal matrices U and V (see the text for the details).

## 2. Experiments

We compared the accuracy and efficiency of our method with existing methods [4], using simulation data (see the text). The accuracy of the solution is compared with the theoretical bound (the KCR lower bound [3]), too.

We also conducted real image experiments, using the images in Fig. 1. We manually selected 100 pairs of corresponding points and computed the fundamental matrix, using different methods. The residual (the minimum of the cost function) and the execution time (sec) are listed in Table 1.

We can see that optimally correction ML and the proposed method converged to the same solution, while SVD corrections of LS and ML resulted in different values with higher residuals. For this data set, the proposed method took longer time than optimally corrected ML.

The reason for the poor performance of CFNS[2] is fully



Figure 1 Real images and 100 corresponding points.

Table 1 The residual and execution time (sec)			
	method	residual	time
	SVD-corrected LS	45.550	.00052
	SVD-corrected ML	45.566	.00642
	Optimally corrected ML	45.378	.00764
	Our method	45.378	.01136
	CFNS [2]	45.378	.01300

investigated in another paper of the authors [5], where a new method called EFNS is proposed and comparative experiments are conducted.

## References

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