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# Automatic Camera Model Selection for Multibody Motion Segmentation

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**Abstract**

We study the problem of segmenting independently moving objects in a video sequence. Several algorithms exist for classifying the trajectories of the feature points into independent motions, but the performance depends on the validity of the underlying camera imaging model. In this paper, we present a scheme for automatically selecting the best model using the geometric AIC *before* the segmentation stage. Using real video sequences, we confirm that the segmentation accuracy indeed improves if the segmentation is based on the selected model.

## 1. Introduction

Segmenting individual objects from backgrounds is one of the most important tasks of video processing. For images taken by a stationary camera, many segmentation algorithms based on background subtraction and interframe subtraction have been proposed. For images taken by a moving camera, however, the segmentation is very difficult because the objects and the backgrounds are both moving in the image.

While most segmentation algorithms combine various heuristics based on miscellaneous cues such as optical flow, color, and texture, Costeira and Kanade [1] presented a segmentation algorithm based only on the image motion of feature points.

Since then, various modifications and extensions of their method have been proposed [3, 6, 10, 13, 15, 16]. Gear [3] used the reduced row echelon form and graph matching. Ichimura [6] applied the discrimination criterion of Otsu [20] and the QR decomposition for feature selection [7]. Inoue and Urahama [10] introduced fuzzy clustering. Incorporating model selection using the geometric AIC [12] and robust estimation using LMedS [22], Kanatani [13, 15, 16] derived segmentation algorithms called *subspace separation* and *affine space separation*. Maki and Wiles [18] and Maki and Hattori [19] used Kanatani's idea for analyzing the effect of illumination on moving objects. Wu, et al. [27] introduced orthogonal subspace decomposition.

To begin the segmentation, the number of independent motions needs to be estimated. This has usually been handled using empirical thresholds. Recently, Kanatani and Matsunaga [17] and Kanatani [15] proposed the use of model selection for this.

For tracking moving feature points, most authors use the Kanade-Lucas-Tomasi algorithm [24]. To improve the tracking accuracy, Huynh and Heyden [5] and Sugaya and Kanatani [23] showed that outlier trajectories can be removed by robust estimation using LMedS [22] and RANSAC [2]. Ichimura and Ikoma [8] and Ichimura [9] introduced nonlinear filtering.

In this paper, we propose a new method for improving the accuracy of Kanatani's subspace separation [13, 15] and affine space separation [16]. According to Kanatani [13, 16], the trajectories of feature points that belong to a rigid object are, under an affine camera model, constrained to be in a 4-dimensional subspace and at the same time in a 3-dimensional affine space in it. If the object is in a 2-dimensional rigid motion, the resulting trajectories are constrained to be in a 3-dimensional subspace or more strongly in a 2-dimensional affine space in it. Theoretically, the segmentation accuracy should be higher if we use stronger constraints. However, it has been pointed out that this is not necessarily true due to the modeling errors of the camera imaging geometry [16].

To cope with this, Kanatani [15, 16, 17] proposed *a posteriori* reliability evaluation using the geometric AIC [12] and the geometric MDL [14]. However, his procedure is based on the assumption that the segmentation is correctly done. In reality, if the final result is rejected as unreliable by Kanatani's method, one cannot tell whether the assumed model was wrong or the segmentation was not correctly done.

In this paper, we introduce model selection *a priori* for choosing the best camera model and the associated space *before* doing segmentation. Using real video sequences, we demonstrate that the segmentation accuracy indeed improves if the segmentation is based on the selected model.

## 2. Trajectory of Feature Points

We track $N$ rigidly moving feature points over $M$ frames and let $(x_{\kappa\alpha}, y_{\kappa\alpha})$ be the image coordinates of the $\alpha$th point in the $\kappa$th frame. We stack all the image coordinates vertically and represent the entire trajectory by the following *trajectory vector*:

$$\boldsymbol{p}_\alpha = (x_{1\alpha}\ y_{1\alpha}\ x_{2\alpha}\ y_{2\alpha} \cdots\ x_{M\alpha}\ y_{M\alpha})^\top. \tag{1}$$

Regarding the $XYZ$ camera coordinate system as the world coordinate system, we fix a 3-D object coordinate system to the moving object. Let $\boldsymbol{t}_\kappa$ and $\{\boldsymbol{i}_\kappa, \boldsymbol{j}_\kappa, \boldsymbol{k}_\kappa\}$ be, respectively, its origin and 3-D orthonormal basis in the $\kappa$th frame. If we let $(a_\alpha, b_\alpha, c_\alpha)$ be the 3-D object coordinates of the $\alpha$th point, its 3-D position in the $\kappa$th frame is

$$\boldsymbol{r}_{\kappa\alpha} = \boldsymbol{t}_\kappa + a_\alpha \boldsymbol{i}_\kappa + b_\alpha \boldsymbol{j}_\kappa + c_\alpha \boldsymbol{k}_\kappa \tag{2}$$

with respect to the world coordinate system.

If an affine camera model (e.g., orthographic, weak perspective, or paraperspective projection) is assumed, the 2-D position of $\boldsymbol{r}_\alpha$ in the image is given by

$$\begin{pmatrix} x_{\kappa\alpha} \\ y_{\kappa\alpha} \end{pmatrix} = \boldsymbol{A}_\kappa \boldsymbol{r}_{\kappa\alpha} + \boldsymbol{b}_\kappa, \tag{3}$$

where $\boldsymbol{A}_\kappa$ and $\boldsymbol{b}_\kappa$ are, respectively, a $2 \times 3$ matrix and a 2-dimensional vector determined by the position and

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orientation of the camera and its internal parameters in the $\kappa$th frame. From eq. (2), we can write eq. (3) as

$$\begin{pmatrix} x_{\kappa\alpha} \\ y_{\kappa\alpha} \end{pmatrix} = \tilde{\boldsymbol{m}}_{0\kappa} + a_\alpha \tilde{\boldsymbol{m}}_{1\kappa} + b_\alpha \tilde{\boldsymbol{m}}_{2\kappa} + c_\alpha \tilde{\boldsymbol{m}}_{3\kappa}, \quad (4)$$

where $\tilde{\boldsymbol{m}}_{0\kappa}, \tilde{\boldsymbol{m}}_{1\kappa}, \tilde{\boldsymbol{m}}_{2\kappa}$, and $\tilde{\boldsymbol{m}}_{3\kappa}$ are 2-dimensional vectors determined by the position and orientation of the camera and its internal parameters in the $\kappa$th frame. From eq. (4), the trajectory vector $\boldsymbol{p}_\alpha$ of eq. (1) can be written in the form

$$\boldsymbol{p}_\alpha = \boldsymbol{m}_0 + a_\alpha \boldsymbol{m}_1 + b_\alpha \boldsymbol{m}_2 + c_\alpha \boldsymbol{m}_3, \quad (5)$$

where $\boldsymbol{m}_0$, $\boldsymbol{m}_1$, $\boldsymbol{m}_2$ and $\boldsymbol{m}_3$, are the $2M$-dimensional vectors obtained by stacking $\tilde{\boldsymbol{m}}_{0\kappa}, \tilde{\boldsymbol{m}}_{1\kappa}, \tilde{\boldsymbol{m}}_{2\kappa}$,and $\tilde{\boldsymbol{m}}_{3\kappa}$ vertically over the $M$ frames, respectively.

## 3. Constraints on Image Motion

Eq. (5) implies that the trajectory vectors of the feature points that belong to the same object are constrained to be in the 4-dimensional subspace spanned by $\{\boldsymbol{m}_0, \boldsymbol{m}_1, \boldsymbol{m}_2, \boldsymbol{m}_3\}$ in $\mathcal{R}^{2M}$. It follows that multiple moving objects can be segmented into individual motions by separating the trajectory vectors $\{\boldsymbol{p}_\alpha\}$ into distinct 4-dimensional subspaces. This is the principle of the *subspace separation* [13, 15].

However, we can also see that the coefficient of $\boldsymbol{m}_0$ in eq. (5) is identically 1 for all $\alpha$. This means that the trajectory vectors are also in the 3-dimensional affine space within that 4-dimensional subspace. It follows that multiple moving objects can be segmented into individual motions by separating the trajectory vectors $\{\boldsymbol{p}_\alpha\}$ into distinct 3-dimensional affine spaces. This is the principle of the *affine space separation* [16].

Theoretically, the segmentation accuracy should be higher if a stronger constraint is used. However, eq. (5) was derived from an affine camera model, while the imaging geometry of real cameras is perspective projection. It can be shown [16] that the modeling errors for approximating the perspective projection by an affine camera are larger for the affine space constraint than for the subspace constraint. In general, the stronger the constraint, the more vulnerable to modeling errors. Conversely, the solution is more robust to modeling errors, if not very accurate, when weaker constraints are used.

According to Kanatani [16], the choice between the subspace separation and the affine space separation depends on the balance between the camera modeling errors and the image noise. The subspace separation performs well when the perspective effects are strong and the noise is small, while the affine space separation performs better for large noise with weak perspective effects. However, we do not know a priori which is the case for a given video sequence.

If the object motion is planar, i.e., if the object merely translates, rotates, and changes the scale within the 2-dimensional image, one of the three vectors $\boldsymbol{m}_1$, $\boldsymbol{m}_2$, and $\boldsymbol{m}_3$ can be set $\mathbf{0}$. Hence, $\boldsymbol{p}_\alpha$ is constrained to be in a 3-dimensional subspace. Since the coefficient of $\boldsymbol{m}_0$ is identically 1, $\boldsymbol{p}_\alpha$ is also in a 2-dimensional affine space within that 3-dimensional subspace. It follows that we can segment multiple planar motions into individual objects by separating the trajectory vectors $\{\boldsymbol{p}_\alpha\}$ into distinct 3-dimensional subspaces or distinct 2-dimensional affine spaces. However, we do not know a priori if the object motion is planar or which constraint should be used for a given video sequence.

## 4. A Priori Camera Models

For simplicity, let us hereafter call the constraint that specifies the camera imaging model and the type of motion the *camera model*. As we have observed, we can expect high accuracy if we know which camera model is suitable and accordingly use the corresponding algorithm. We may test all the models and the associated segmentation methods and evaluate the reliability of the results *a posteriori*, as Kanatani suggested [15, 16, 17]. However, this works only if the segmentation is done correctly; if the final result is rejected as unreliable, one cannot tell whether the assumed model was wrong or the segmentation was not correctly done.

To overcome this difficulty, we introduce camera models that should be valid *irrespective* of the segmentation results. If, for example, one object is moving relative to a stationary background while the camera is moving, two independent motions are observed in the image: the object motion and the background motion. Since the trajectory vectors for each motion is in a 4-dimensional subspace or a 3-dimensional affine space in it, the entire trajectory vectors $\{\boldsymbol{p}_\alpha\}$ should be in an 8-dimensional subspace $\mathcal{L}^8$ or a 7-dimensional affine space $\mathcal{A}^7$ in it[1] .

If the object motion and the background motion are both planar, the trajectory vectors for each motion are in a 3-dimensional subspaces or a 2-dimensional affine spaces in it, so the entire trajectory vectors $\{\boldsymbol{p}_\alpha\}$ should be in a 6-dimensional subspace $\mathcal{L}^6$ or a 5-dimensional affine space $\mathcal{A}^5$ in it.

It follows that in the pre-segmentation stage we have $\mathcal{L}^8$, $\mathcal{A}^7$, $\mathcal{L}^6$, and $\mathcal{A}^5$ as candidate models *irrespective* of the segmentation results. If the number of independent motions is $m$, they are replaced by $\mathcal{L}^{4m}$, $\mathcal{A}^{4m-1}$, $\mathcal{L}^{3m}$, and $\mathcal{A}^{3m-1}$, respectively.

## 5. Model Selection

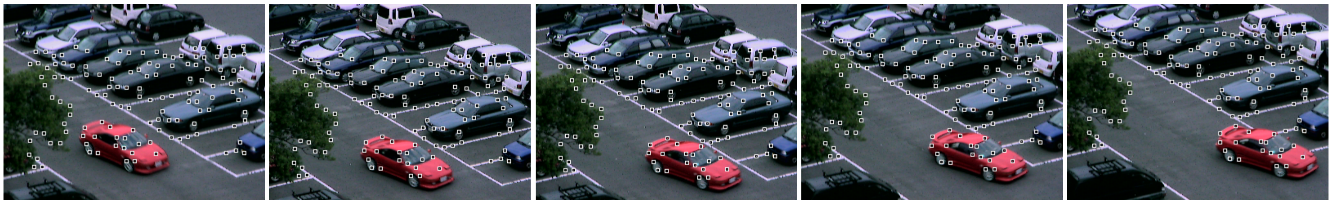
A naive idea for model selection is to fit the candidate models to the observed data and choose the one for which the *residual*, i.e., the sum of the square distances of the data points to the fitted model, is the smallest. This does not work, however, because the model that has the largest degree of freedom, i.e., the largest number of parameters that can specify the model, always has the smallest residual. It follows that we must balance the increase in the residual against the decrease in the degree of freedom. For this purpose, we use the geometric AIC [11, 12] (see [25, 26] for other criteria).

Let $n = 2M$. For the $N$ trajectory vectors $\{\boldsymbol{p}_\alpha\}$ in an $n$-dimensional space, define the $n \times n$ *moment matrix* by

$$\boldsymbol{M} = \sum_{\alpha=1}^{N} \boldsymbol{p}_\alpha \boldsymbol{p}_\alpha^\top. \quad (6)$$

Let $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ be its eigenvalues. If we optimally fit a $d$-dimensional subspace to $\{\boldsymbol{p}_\alpha\}$, the

[1] The minimal subspace that includes an $n_1$-dimensional subspace and an $n_2$-dimensional subspace has dimension $n_1 + n_2$, while the minimal affine space that includes an $m_1$-dimensional affine space and an $m_2$-dimensional affine space has dimension $m_1 + m_2 + 1$.



| Model | $\mathcal{L}^8$ | $\mathcal{A}^7$ | $\mathcal{L}^6$ | $\mathcal{A}^5$ |
|---|---|---|---|---|
| G-AIC | 836.9 | 779.1 | 688.9 | **631.1** |

| Method | C-K | ICH | S-M | $\mathcal{L}^8$ | $\mathcal{A}^7$ | $\mathcal{L}^6$ | $\mathcal{A}^5$ |
|---|---|---|---|---|---|---|---|
| Correctness (%) | 85.3 | 92.6 | 86.8 | 75.0 | 86.0 | 97.7 | **100** |

Figure 1: Upper: Input video sequence (1st, 8th, 15th, 22th, 30th frame) and successfully tracked 136 feature points. Lower: The geometric AIC for each model (left); the correctness of segmentation for different methods (right).

resulting residual $J_{\mathcal{L}^d}$ is given by

$$J_{\mathcal{L}^d} = \sum_{i=d+1}^{n} \lambda_i. \tag{7}$$

The geometric AIC has the following form [11, 12]:

$$\text{G-AIC}_{\mathcal{L}^d} = J_{\mathcal{L}^d} + 2d(N+n-d)\epsilon^2. \tag{8}$$

Here, $\epsilon$, which we call the *noise level*, is the standard deviation of the noise in the coordinates of the feature points.

For fitting a $d$-dimensional affine space to $\{\boldsymbol{p}_\alpha\}$, the geometric AIC is computed as follows. Define the $n \times n$ moment matrix matrix by

$$\boldsymbol{M}' = \sum_{\alpha=1}^{N} (\boldsymbol{p}_\alpha - \boldsymbol{p}_C)(\boldsymbol{p}_\alpha - \boldsymbol{p}_C)^\top, \tag{9}$$

where $\boldsymbol{p}_C$ is the centroid of $\{\boldsymbol{p}_\alpha\}$. Let $\lambda'_1 \geq \lambda'_2 \geq \cdots \geq \lambda'_n$ be the eigenvalues of the matrix $\boldsymbol{M}'$. The residual $J_{\mathcal{A}^d}$ of fitting a $d$-dimensional affine space to $\{\boldsymbol{p}_\alpha\}$ is given by

$$J_{\mathcal{A}^d} = \sum_{i=d+1}^{n} \lambda'_i. \tag{10}$$

The geometric AIC has the following form [11, 12]:

$$\text{G-AIC}_{\mathcal{A}^d} = J_{\mathcal{A}^d} + 2\Big(dN + (d+1)(n-d)\Big)\epsilon^2. \tag{11}$$

We compare the geometric AIC for each candidate model and choose the one that has the smallest geometric AIC.

## 6. Real Video Experiments

We tested our proposed method using real video sequences. The image size is 320 × 240 pixels. The number of independent motions can be estimated by the method described in [17, 15]. In our experiments, however, we assumed that the number of independent motions was two in order to focus only on the segmentation performance of our proposed model selection. We first removed outlier trajectories by the method described in [23] and applied the model selection to the resulting outlier-free trajectories.

Fig. 1 shows five frames decimated from a 30 frame sequence taken by a moving camera. We correctly tracked 136 points, which are indicated by the symbol □ in the images.

We fitted to them an 8-dimensional subspace $\mathcal{L}^8$, a 7-dimensional affine space $\mathcal{A}^7$, a 6-dimensional subspace $\mathcal{L}^6$, and a 5-dimensional affine space $\mathcal{A}^5$ and computed their geometric AICs. The lower left table lists their values. As we can see, the 5-dimensional affine space $\mathcal{A}^5$ was chosen as the best model.

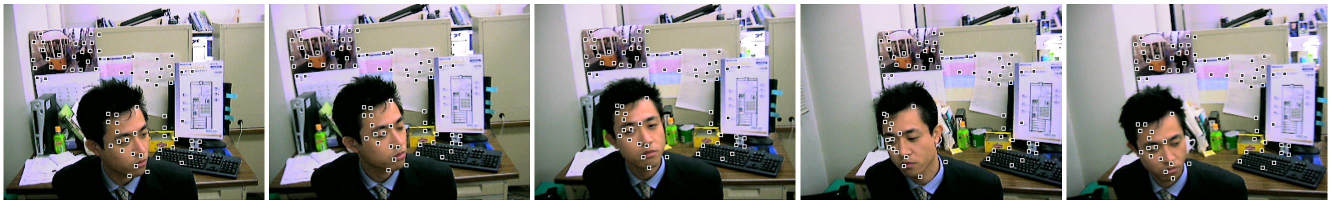
In order to compute the geometric AIC as given in eqs. (8) and (11), we need to know the noise level $\epsilon$. Theoretically, it can be estimated from the residual of the most general model $\mathcal{L}^8$ if the noise in each frame is independent and Gaussian [11]. In reality, however, strong correlations exist over consecutive frames, so that some points are tracked unambiguously throughout the sequence, while others fluctuate from frame to frame [23]. Considering this, we empirically set $\epsilon$ to 0.5 pixels[2] . We have confirmed that changing this value over 0.1 ∼ 1.0 does not affect the selected model in this and the subsequent experiments.

This video sequence was taken from a distance, and the object (a car) and the background are moving almost rigidly in the image. Hence, the selection of $\mathcal{A}^5$ seems reasonable. The lower right table compares the correctness of segmentation measured by (the number of correctly classified points)/(the total number of points) in percentage for different methods. The correctness of individual matches was judged by visual inspection.

In the table, "C-K" means the method of Costeira and Kanade [1], which progressively interchanges the rows and columns of the (shape) interaction matrix to make it approximately block-diagonal in such a way that the off-diagonal elements have small absolute values; "ICH" means the method of Ichimura [6], who applied the *Otsu discrimination criterion* [20] to each row of the interaction matrix and segmented the elements according to the row with the highest discrimination measure; "S-M" indicates the result obtained by partitioning the graph defined by the interaction matrix (the feature points as vertices and the absolute values of its elements as the weights of the corresponding edges) in such a way that the *normalized cut* [21] is minimized. The fuzzy clustering of Inoue and Urahama [10] is also based on a similar idea. The symbols $\mathcal{L}^8$, $\mathcal{A}^7$, $\mathcal{L}^6$, and $\mathcal{A}^5$ indicate the subspace separation and affine space separation using the corresponding models. As expected, the affine space separation using the selected model $\mathcal{A}^5$ alone achieved 100% correct segmentation.

Fig. 2 shows a different sequence, through which 73

[2] We also used this value for the outlier removal procedure [23].



| Model | $\mathcal{L}^8$ | $\mathcal{A}^7$ | $\mathcal{L}^6$ | $\mathcal{A}^5$ |
|---|---|---|---|---|
| G-AIC | **2117.9** | 2281.5 | 3158.5 | 3340.1 |

| Method | C-K | ICH | S-M | $\mathcal{L}^8$ | $\mathcal{A}^7$ | $\mathcal{L}^6$ | $\mathcal{A}^5$ |
|---|---|---|---|---|---|---|---|
| Correctness (%) | 76.7 | 58.9 | 76.7 | **93.1** | 60.2 | 57.5 | 89.0 |

Figure 2: Upper: Input video sequence (1st, 25th, 50th, 75th, 100th frame) and successfully tracked 73 feature points. Lower: The geometric AIC for each model (left); the correctness of segmentation for different methods (right).

points are tracked over 100 frames. This sequence was taken near the moving object (a person) by a moving camera, so the perspective effects are relatively strong. As expected, the 8-dimensional subspace $\mathcal{L}^8$ was chosen as the best model, and the subspace separation using it gave the best result.

The reason why the subspace separation did not achieve 100% correct segmentation seems to be that the method is based on the affine camera model, although the modeling error is smaller than for the affine space separation. In fact, we observed that the accuracy unexpectedly decreased as we increased the number of the internally used LMedS iterations to impose the subspace constraint very strictly.

## 7. Concluding Remarks

We have proposed a technique for automatically selecting the best model by using the geometric AIC in an attempt to improve the segmentation accuracy of the subspace separation [13] and the affine space separation [16] *before* doing segmentation. Using real video sequences, we demonstrated that the separation accuracy indeed improves if the segmentation is based on the selected model.

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