implies.

Reference

6) Ishihara, K., Tatsuoka, F. and Yasuda, S. (1975): "Undrained deformation and liquefaction of sand under cyclic stresses," Soils and Foundations, Vol. 15, No. 1, pp. 29-44.

STRESS-STRAIN RELATIONSHIP OF SOILS AS ANISOTROPIC BODIES UNDER THREE DIFFERENT PRINCIPAL STRESSES*

Discussion by KEN-ICHI KANATANI**

The author asserts that the dilatancy of soil is a phenomenon caused by *anisotropy*. The purpose of this discussion is to point out that the author's formulation is for a nonlinear *isotropic* materials.

1. *What is dilatancy and shearing stress?*

The dilatancy of soil is usually regarded as the volume change due to *shearing stress*. What is shearing stress exactly? From the viewpoint of mathematics, the shearing stress is nothing but the *deviatoric part* of the stress tensor. The stress tensor σ is resolved into the scalar part and the deviatoric part as

$$\sigma_{ij}=-p\delta_{ij}+\tilde{\sigma}_{ij}, \tag{24}$$

where $\tilde{\sigma}$ and p are respectively the stress deviator and the hydrostatic pressure defined by

$$\tilde{\sigma}_{ij}\equiv\sigma_{ij}-(\sigma_{kk}/3)\delta_{ij}, \quad p\equiv-\sigma_{kk}/3 \tag{25}$$

The summation convention is adopted, and δ_{ij} is the Kronecker delta. This definition of shearing stress by $\tilde{\sigma}$ is the only one that is invariant to coordinate transformations (e. g., Kanatani, 1980). The author claims that this is his new idea, but it is a well established fact in continuum mechanics.

2. *Dilatancy does not occur in a linear material whether it be isotropic or anisotropic*

As is well known, the constitutive equation of a linear isotropic material is

$$e_{ij}=[\sigma_{ij}-(\nu/(1+\nu))\sigma_{kk}\delta_{ij}]/2\mu, \tag{26}$$

where μ is the shear modulus, ν the Poisson ratio and e the strain tensor. If we take the scalar part and the deviatoric part of the both sides of Eq. (26), we obtain

$$v=-p/\kappa, \quad \tilde{e}_{ij}=\tilde{\sigma}_{ij}/2\mu, \tag{27}$$

where $v\equiv e_{kk}$ is the rate of volume increase, $\kappa=(1-2\nu)/2\mu(1+\nu)$ is the bulk modulus and $\tilde{e}$ is the strain deviator, i.e., the shearing strain (e. g., Kanatani, 1980). The volume change v is determined by the hydrostatic pressure p alone, and the shearing strain $\tilde{e}$ is

* By Shōji Yoshida, Vol. 20, No. 1, Mar. 1980, pp. 1-11.

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determined by the shearing stress $\tilde{\sigma}$ alone. Hence, dilatancy does not occur in a linear isotropic material. Then, what about a linear *anisotropic* material?

If the material is anisotropic, then non-zero volume increase v ($\neq 0$) may possibly be observed under shearing stress $\tilde{\sigma}$. If it is the case, however, the linearity implies that the volume increase by $-v$ should be observed under $-\tilde{\sigma}$, i. e., the shearing in the opposit direction (Fig. 10). In other words, $v(\tilde{\sigma})$ is a linear function as is schematically described in Fig. 10. This phenomenon is not what is usually called "dilatancy". It is merely an anisotropic deformation.

The dilatancy actually observed is schematically shown in Fig. 11, and thus it is essentially a *nonlinear* phenomenon. If the material is isotropic, then $v_1=v_2$ and the curve $v(\tilde{\sigma})$ is symmetric with respect to the v-axis. If it is anisotropic, then $v_1 \neq v_2$ in general and the curve $v(\tilde{\sigma})$ is nonsymmetric in general.

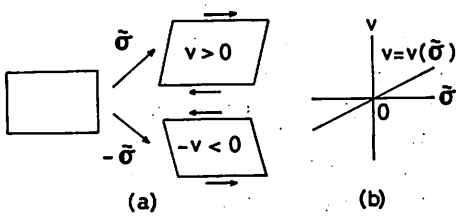

Fig. 10. (a) Linear anisotropic deformation (b) Linear volumetric strain relation

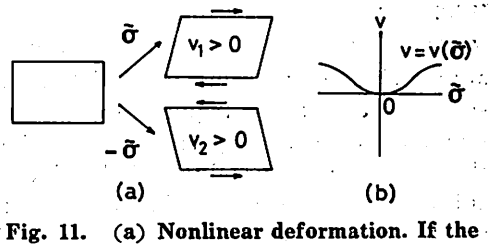

Fig. 11. (a) Nonlinear deformation. If the material is isotropic, then $v_1=v_2$. (b) Dilatancy relation. If the material is isotropic, the curve is symmetric with respect to the v-axis.

3. *How can the nonlinear isotropy be described?*

The material is isotropic *if and only if the strain* $\boldsymbol{e}$ *is determined by the present stress* $\boldsymbol{\sigma}$ *alone*, i. e., $\boldsymbol{e}=\boldsymbol{e}(\boldsymbol{\sigma})$. (Here, we do not consider rheology and history dependence.) According to the Hamilton-Cayley theorem of linear algebra, the powers $\boldsymbol{\sigma}^3, \boldsymbol{\sigma}^4, \boldsymbol{\sigma}^5, \cdots\cdots$ are all expressed in terms of $\boldsymbol{\sigma}, \boldsymbol{\sigma}^2$ and the three scalar invariants J_1, J_2 and J_3 of $\boldsymbol{\sigma}$. Thus, the most general constitutive equation is

$$e_{ij}=\alpha_1\delta_{ij}+\alpha_2\sigma_{ij}+\alpha_3\sigma_{ik}\sigma_{kj}, \tag{28}$$

where α_1, α_2 and α_3 are scalar functions of the three invariants J_1, J_2 and J_3. From this, we can conclude that the principal stress axes coincide with those of strain. Taking the common principal axes as the coordinate system and numbering the axes so that $\sigma_1 \geq \sigma_2 \geq \sigma_3$, we can express Eq. (28) in the form of

$$e_1=e_1(\sigma_1, \sigma_2, \sigma_3), \quad e_2=e_2(\sigma_1, \sigma_2, \sigma_3), \quad e_3=e_3(\sigma_1, \sigma_2, \sigma_3). \tag{29}$$

Determination of the three function forms for e_1, e_2 and e_3 is equivalent to determining the three function forms for α_1, α_2 and α_3 in Eqs. (28). Thus, any material whose constitutive relation is expressed in the form of Eqs. (29) is *necessarily isotropic*. Indeed, we are claiming that Eqs. (29) are valid *no matter how the principal axes are directed relative to the material.*

If the material is anisotropic, we must have

$$\boldsymbol{e}=\boldsymbol{e}(\boldsymbol{\sigma}, \boldsymbol{C}), \tag{30}$$

where $\boldsymbol{C}$ is a tensor intrinsically determined *by the material itself (not by the stress)*. Hence, the principal stress axes do not necessarily coincide with those of stress, and we cannot have expressions like Eqs. (29). If the material has orthogonal anisotropy,

and if the external loading happens to be such that the principal stress axes coincide with the material axes of anisotropy, we can write down the results in the form of Eqs. (29). However, they do not describe the constitutive relation because they do not hold for a general stress state specified by the principal components σ_1, σ_2 and σ_3. The constitutive equation must necessarily be given by the form of Eq. (30).

4. *Conclusions*

(1) The author's results are expressed in the form of Eqs. (29), i.e., the strain is completely determined by the principal stress components σ_1, σ_2 and σ_3 alone without any reference to the material axes of anisotropy. In other words, the author is claiming that his results hold for any stress state which has σ_1, σ_2 and σ_3 as the principal components. This means that his formulation is for a (nonlinear) *isotropic* material. (2) The author assumes that the axes of the internal soil structure, which he calls "anisotropy", are not fixed by the material itself but *are determined by and coincide with the principal stress axes.* This is nothing but the very *criterion of isotropy itself.* (3) The author obtained the numerical data by triaxial compression tests. The triaxial compression test essentially requires the assumption of isotropy. For example, the principal strain axes are usually assumed to coincide with the principal stress axes. Moreover, the equipment is usually cylindrical or cubic, not spherical. Hence, some amount of anisotropy is always introduced due to the shape of the equipment. This anisotropy is not the pure characteristics of the material but rather those of the equipment. In order to distinguish the anisotropy induced by the equipment from that of the material, one must rotate the material inside the equipment to test whether the material axes of anisotropy exist or not. But again the author does not refer to the material axes of anisotropy at all.

In view of these facts altogether, it is concluded that the author's formulation is for a *nonlinear isotropic material.*

Reference

13) Kanatani, K. (1980): "Shear, compression and constitutive equation," Tsuchi-to-Kiso, JSSMFE, Vol. 28, No. 12, pp. 5-9 (in Japanese).

A STRESS-STRAIN RELATIONSHIP OF NORMALLY CONSOLIDATED COHESIVE SOIL UNDER GENERAL STRESS CONDITION*

Discussion by V. K. Tokhi**

The writer appreciates the work of Ohmaki in developing a stress-strain relationship which is applicable to normally consolidated soils under general stress system from parameters which are obtained solely from triaxial test. The writer wishes to make a contri-

* By Seiki Ohmaki, Vol. 20, No. 1, Mar. 1980, pp. 29-43.

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