(A), (B) and (C) are separately calculated, and are summed up to give the final crack tensor F_{ij}, as follows :

$$F_{ij}=F_{ij}^{(A)}+F_{ij}^{(B)}+F_{ij}^{(C)} =\begin{bmatrix} 9.965 & 0.1616 & 1.651 \\ 0.1616 & 5.691 & 1.066 \\ 1.651 & 1.066 & 7.715 \end{bmatrix} \quad (53)$$

Elastic compliance $\bar{C}_{ijkl}$ due to the presence of joints can be estimated by substituting Eq. (53) in Eq. (23) :

$$\bar{C}_{ijkl}=\frac{1}{4D}(\delta_{il}F_{jk}+\delta_{jl}F_{ik}+\delta_{jk}F_{il}+\delta_{ik}F_{jl}) \quad (23)$$

It must be emphasized, however, that Eq. (23) gives an approximation of $\bar{C}_{ijkl}$ for the granite if only joints are entirely open.

The observed joints are more or less contaminated with clay minerals which stem from weathering of the host rock. The clay minerals have high compressibility as well as extremely low shear strength. This seems to support the idea that joints can be treated as a open cracks. It is also true, however, that some spots on a joint surface are left unweathered where stresses are effectively transmitted. Since joints are considered partially open, then a correction of Eq. (23) is carried out ;

$$\bar{C}_{ijkl}=\frac{\alpha}{4D}(\delta_{il}F_{jk}+\delta_{jl}F_{ik}+\delta_{jk}F_{il}+\delta_{ik}F_{jl}) \quad (54)$$

In order to evaluate the correction coefficient α, an additional information is necessary (Oda and Maeshibu, 1984).

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PRINCIPLES FOR CONSTITUTIVE EQUATIONS AND EXPRESSIONS OF ANISOTROPY IN SOIL MATERIALS*

Discussion by KEN-ICHI KANATANI**

1. *Isotropy*

The authors write that Kanatani (1982) "seems to confuse the isotropy with the objectivity." As the authors state in the paper, the distinction between them is very subtle, and it is sometimes very difficult to discuss them separately. Moreover, there are many different ways of defining them. However, there exists a fundamental difference, namely the "isotropy" is an "assumption" about the material in question, while the "objectivity" is a "principle" applicable irrespective of the material under consideration.

For example, suppose we want to obtain a constitutive equation which describes the stress $\boldsymbol{\sigma}$ in terms of the strain $\boldsymbol{e}$. We naturally expect an equation of the form $\boldsymbol{\sigma}=\boldsymbol{f}(\boldsymbol{e})$, where $\boldsymbol{f}(.)$ has a coordinate independent function form. Suppose stress $\boldsymbol{\sigma}$ is observed as an output to strain $\boldsymbol{e}$ as an input by an experiment. In this case, we expect to observe stress $\boldsymbol{R\sigma R}^T$ as an output to strain $\boldsymbol{ReR}^T$, where $\boldsymbol{R}$ is a rotation matrix. If it turns out that this does not hold, then we cannot obtain the form of $\boldsymbol{\sigma}=\boldsymbol{f}(\boldsymbol{e})$. On the other hand, if the material is such that

$$\boldsymbol{R\sigma R}^T=\boldsymbol{f}(\boldsymbol{ReR}^T), \quad (60)$$

always holds, where $\boldsymbol{f}(.)$ is the "input-output response" of the experiment described with respect to a fixed coordinate system (i. e., not yet a definite function form), then we can obtain a constitutive equation of the form $\boldsymbol{\sigma}=\boldsymbol{f}(\boldsymbol{e})$ with $\boldsymbol{f}(.)$ as a coordinate independent function form. If this "materi-

* By Chikayoshi Yatomi and Akira Nishihara, Vol. 24, No. 3, September 1984, pp. 15-26.

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al property" is satisfied, the "input-output response" can be called "isotropic." Eq. (60) states that the output to the rotated input equals rotated output to the original input. In other words, the field does not have any directionality except $\boldsymbol{e}$, or equivalently when $\boldsymbol{e}=\mathbf{0}$ (i. e., existence of an "isotropic reference state").

Thus, explanation of Kanatani (1982) goes like this. If Eq. (60) is satisfied by the material in question (i. e., this is an "assumption" about that material), then the material is "isotropic" and we can obtain a constitutive equation of an invariant form $\boldsymbol{\sigma}=\boldsymbol{f}(\boldsymbol{e})$. In contrast, the authors seem to explain that if we have a constitutive equation of an invariant form $\boldsymbol{\sigma}=\boldsymbol{f}(\boldsymbol{e})$, then Eq. (60) must be satisfied due to "objectivity." Stated in this way, the reasoning no longer has anything to do with a particular material, and it becomes a universal statement.

It seems to the writer that both are saying essentially the same thing but from a different point of view or by a different chain of definitions. The writer does not know which way (or any other one) is really "orthodox," but he realizes that there may as well exist many different ways of reasoning to explain essentially the same thing.

Kanatani (1982) further goes on, saying that suppose Eq. (60) is not satisfied but instead, by introducing another quantity $\boldsymbol{D}$, the "fabric tensor," the "input-output response" of the experiment is found to satisfy

$$\boldsymbol{R\sigma R}^T=\boldsymbol{f}(\boldsymbol{ReR}^T, \boldsymbol{RDR}^T), \qquad (61)$$

then we have a constitutive equation of an invariant form $\boldsymbol{\sigma}=\boldsymbol{f}(\boldsymbol{e}, \boldsymbol{D})$. In other words, if the "material" is such that Eq. (61) is satisfied, then we obtain $\boldsymbol{\sigma}=\boldsymbol{f}(\boldsymbol{e}, \boldsymbol{D})$, where $\boldsymbol{f}(.,\ .)$ is an isotropic function. Eq. (61) states that if we take $\boldsymbol{e}$ and $\boldsymbol{D}$ away (or setting $\boldsymbol{e}=\mathbf{0}$, $\boldsymbol{D}=\boldsymbol{I}$) no quantity is left to the material (i. e., the "reference state") that characterizes directionality. The authors may instead claim that if we have a definite constitutive equation $\boldsymbol{\sigma}=\boldsymbol{f}(\boldsymbol{e}, \boldsymbol{D})$ in the first place, then Eq. (61) is satisfied due to "objectivity."

2. *Objectivity*

It seems that the authors are presenting more fundamental and general discussions, namely precise description of the objectivity, which implicitly underlies Kanatani's arguments. The reason why he did not discuss the objectivity is that he thought that it had already been well known and that the discussion given there was almost sufficient for practical purposes. The authors assert that existing studies are only for finite deformations and that even the definition of a simple anisotropy is not clear in infinitesimal theories. This is hardly believable, since the subject has an old history and many mathematicians has been devoted to it. Therefore, the sort of discussions given by the authors should be known even though no specific statements are found in literature.

As the authors point out, the objectvity is a general "principle" which does not depend on specific materials and it gives a rigorous basis "not" to obtain unrealistic constitutive equations. However, equally important is consideration of specific "material properties" of the material in question. The writer fears that the authors' strong emphasis on the objectivity might mislead some people in soil mechanics who are not accustomed to this sort of discussion.

3. *Reference*

The authors describe anisotropy in the form of $\boldsymbol{\sigma}=\boldsymbol{f}(\boldsymbol{e}, \boldsymbol{K})$, where K is what the authors call the "reference tensor." Then, they regard $\boldsymbol{K}$ to be a function of, say, the stress history $\boldsymbol{\sigma}_{-s}$ and the reference tensor $\boldsymbol{K}'$ at a previous time in the form of $\boldsymbol{K}=\boldsymbol{K}(\boldsymbol{\sigma}_{-s}, \boldsymbol{K}')$, and classify inherent, induced and combined anisotropies. (The notation is slightly changed here for convenience.)

The writer wonders what $\boldsymbol{\sigma}=\boldsymbol{f}(\boldsymbol{e}, \boldsymbol{K})$ means. Is $\boldsymbol{K}$ an argument of function $\boldsymbol{f}(.,.)$ which can take on arbitrary tensor values? Do they mean that the time change of the material due to deformation history is given by only varying $\boldsymbol{K}$ with the same function form $\boldsymbol{f}(.,\ .)$? In other words, does the deformation history enter only in the form of changing $\boldsymbol{K}$? If that is the case, the authors are assuming a very restricted, very special type of materials and material symmetries. This is in general not always possible.

If certain requirements are satisfied, we may obtain that type of constitutive equations but we need in general an infinite series of tensors, which characterize the "state" of the material (e.g., see Onat (1982a, 1982b)). Generally speaking, the "state" of a material is an abstract concept of anything that might affect subsequent mechanical properties of that material including all the past history of deformation. Hence, if we happen to be able to characterize the "state" by a finite number of vectors and tensors, then we can obtain a constitutive equation which has a fixed function form, yet can describe all effects of deformation histories. For example, if the "state" is characterized by a distribution of some directional data, it may be specified by a series of "fabric tensors" $D_{ij}, D_{ijkl}, D_{ijklmn}, \cdots$ (cf. Kanatani (1984)).

If the authors' "reference tensor" or "reference vector" means the "state," then their replacement of $\boldsymbol{\sigma}=\boldsymbol{f}(\boldsymbol{e})$ by $\boldsymbol{\sigma}=\boldsymbol{f}(\boldsymbol{e}, \boldsymbol{b})$ or $\boldsymbol{\sigma}=\boldsymbol{f}(\boldsymbol{e}, \boldsymbol{K})$ with $\boldsymbol{f}(.,.)$ a fixed function form is a very strong assumption, assigning a special type of material symmetry to the material in question. The problem of how the "state" is described, namely, what vectors and tensors we should incorporate is not so simple, yet is crucial to subsequent formulations. This is one of the most difficult problems in the study of real materials. The writer believes that due emphasis should be placed on this problem, which is the very crux of soil mechanics today.

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PRINCIPLES FOR CONSTITUTIVE EQUATIONS AND EXPRESSIONS OF ANISOTROPY IN SOIL MATERIALS*

Closure by CHIKAYOSHI YATOMI** and AKIRA NISHIHARA***

The writers sincerely thank Professor Kanatani for his stimulating discussions.

(1), (2) Isotropy and Objectivity

Main cause for the statement appeared in the author's paper that Kanatani (1982) "seems to confuse the isotropy with the objectivity" came from the definition of isotropy by isotropic functions without any explicit reference of objectivity and any clear statement about the dependency of the form of $\boldsymbol{F}$ on the reference state in the discusser's original paper.

The discusser considered a constitutive equation such as

$$d\boldsymbol{\sigma}=\boldsymbol{F}(d\boldsymbol{e}, \boldsymbol{D})$$

with $(d\boldsymbol{\sigma}, d\boldsymbol{e}, \boldsymbol{D})=(\boldsymbol{0}, \boldsymbol{0}, \boldsymbol{1})$ at the initial state, where $\boldsymbol{D}$ was called "Fablic tensor" and its principal axes were the directions of the axes of symmetry of all the states. The discusser then concluded from the defini-

* Vol. 24, No. 3, September 1984, pp. 15-26. (Previous discussion by Ken-ichi Kanatani, Vol. 25, No. 2, pp. 167-169.)

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tion that "$\boldsymbol{F}$ must be an isotropic function of de and $\boldsymbol{D}$, since the material is isotropic". Since the reference state was taken at the initial state, $\boldsymbol{D}=1$ at the initial state shows no directional characteristic in the reference state. The material above is, therefore, automatically isotropic whatever the function $\boldsymbol{F}$ is. On the other hand, the objectivity then requires the $\boldsymbol{F}$ to be an isotropic function of de and $\boldsymbol{D}$. We should say, therefore, that "$\boldsymbol{F}$ must be an isotropic function of de and $\boldsymbol{D}$ since the objectivity must be satisfied". Even if $\boldsymbol{D}\neq 1$ at the initial state, that is, the material is initially anisotropic, $\boldsymbol{F}$ must be an isotropic function of de and $\boldsymbol{D}$ from the objectivity.

Refer to the J. Casey and Naghdi (1981) and the references cited there for the discussions on the objectivity in infinitesimal theories.

(3) *Reference State*

The "reference state" is here employed only to discuss the material symmetry of solids; that is, isotropy and anisotropy of mechanical response of solids. To check the symmetry, all we have to do is making experiments of the mechanical responses with changing the orientation of the reference body. Thus the difference of the reference state is then nothing but the difference of the orientation of a rigid body.

In order to determine the orientation of a rigid body, it is sufficient to have three (one) independent scalar parameters in a three (two) dimensional space, such as Eulerian angle. Since the Eulerian angle was, however, not necessarily convenient for our purpose, we introduced here three independent (orthogonal) unit vectors $\boldsymbol{b}^{\alpha}$ ($\alpha=1, 2, 3$) (Eq. 23) fixed in the undeformed (reference) state, which were called "reference vectors". Note that three (two) independent unit vectors fixed in a rigid body have exactly three (one) independent components with respect to a coordinate system fixed in a three (two) dimensional space. The three (two)$\boldsymbol{b}^{\alpha}$ are, therefore, sufficient to express the most general anisotropic materials in the three (two) dimensional responce. The "reference tensor $\boldsymbol{K}$" was employed here just to express orthotropic anisotropy, since we considered that the orthotropy was enough to discuss the anisotropy of soils, in particular, the stress induced anisotropy. An "infinite series of vectors and tensors" is, therefore, redundant for out purpose. Any nonzero vectors (in a three dimensional space) are linear combination of three reference vectors $\boldsymbol{b}^{\alpha}$ and any order "fabric tensors" $D_{ij}, D_{ijkl}, \cdots\cdots$ (in a three dimensional space) can be represented by three reference vectors $\boldsymbol{b}^{\alpha}$;

$$D_{ij}=\sum_{\alpha,\beta=1}^{3} d_{\alpha\beta}b_i^{\alpha}b_j^{\beta}$$

$$D_{ijkl}=\sum_{\alpha\beta\gamma\delta=1}^{3} d_{\alpha\beta\gamma\delta}b_i^{\alpha}b_j^{\beta}b_k^{\gamma}b_l^{\delta}$$

$$\cdots\cdots\cdots\cdots\cdots$$

where $d_{\alpha\beta}, d_{\alpha\beta\gamma\delta}, \cdots$ are constant.

In order to show that such reference vectors are sufficient to express general anisotropic responses, we give simple examples in two-dimensional response. Consider special cases of

$$\boldsymbol{\sigma}=\hat{\boldsymbol{\sigma}}(\boldsymbol{\varepsilon}, \boldsymbol{b}^{\alpha}) \quad \alpha=1, 2$$

$$\boldsymbol{\sigma}=\hat{\boldsymbol{\sigma}}(\boldsymbol{\varepsilon}, \boldsymbol{K}) \quad \boldsymbol{K}=c_1\boldsymbol{b}^1\otimes\boldsymbol{b}^1+c_2\boldsymbol{b}^2\otimes\boldsymbol{b}^2 \quad (c_1\neq c_2) \tag{62}$$

such as

$$\boldsymbol{\sigma}=f(\boldsymbol{b}^1\cdot\boldsymbol{\varepsilon}\boldsymbol{b}^1, \boldsymbol{b}^2\cdot\boldsymbol{\varepsilon}\boldsymbol{b}^2)\boldsymbol{\varepsilon}$$

$$\boldsymbol{\sigma}=g(\boldsymbol{K}\cdot\boldsymbol{\varepsilon})\boldsymbol{\varepsilon} \tag{63}$$

respectively, where f and g are scalar functions. These are peculiar cases of anisotropic elastic materials, in which stress and strain are coaxial (but not isotropic). Let σ_i, ε_i ($i=1, 2$) be the principal components with principal directions $\boldsymbol{d}^{\alpha}$. A length OA in Figs. 4 and 5 shows only the principal stress σ_1 for given constant principal strains ε_1 and ε_2 ($\varepsilon_1\neq\varepsilon_2$) with a fixed reference vectors $\boldsymbol{b}^{\alpha}$ in the body. (Since $\boldsymbol{\varepsilon}=\varepsilon_1\boldsymbol{d}^1\otimes\boldsymbol{d}^1+\varepsilon_2\boldsymbol{d}^2\otimes\boldsymbol{d}^2$, and therefore, $\hat{\boldsymbol{\sigma}}$ in Eq. (62) are, for a fixed $\boldsymbol{b}^{\alpha}$, even functions of $\boldsymbol{d}^{\alpha}$ for all directions, the directional dependence of σ_1 is symmetric with respect to the origin in Figures.) It is easy to check Eq. (63) satisfy the principle of objectivity. Since f and g in Eq. (63) are invariant under a change

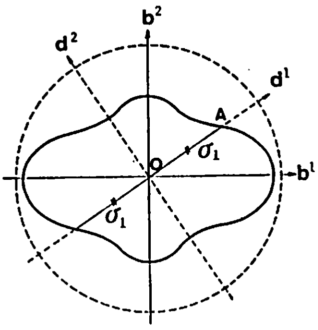

Fig. 4.

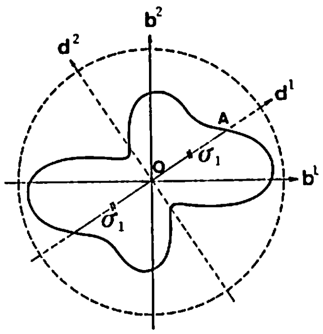

Fig. 5.

of sign of b^α, materials in Eq. (63) are at most orthotropic with the axes of symmetry b^α (Fig. 4). If the response of the material has no symmetry except the origin such as Fig. 5, there is no particularly convenient choice of b^α. We may express such an anisotropic response by introducing $b^1 \cdot \varepsilon b^2$ into f in Eq. $(63)_1$ although b^α have then no prefered directions.

Errata

Finally, we would like to correct here some misprints and mistakes in our paper.

p. 16, in Mathematical Symbols :
$\|A\| = \sqrt{tr(A)^2} \longrightarrow \sqrt{tr(AA^T)}$

p. 18, left line 11 and Fig. 3 :
$\frac{\partial x}{\partial X}$ and $\frac{\partial \bar{x}}{\partial X} \longrightarrow \frac{\partial x_s}{\partial X}$ and $\frac{\partial \bar{x}_s}{\partial \bar{X}}$

p. 19, left line 16 :
Conversly $\longrightarrow$ Conversely

p. 19, right line 20 : 1980 $\longrightarrow$ 1981

p. 20, right last line :
in the anisotropic $\longrightarrow$ in an isotropic

p. 21, left line 7 : strech $\longrightarrow$ stretch

p. 21, right last line : $t=0 \longrightarrow t=t_0$

p. 22, right line 2 from the last :
$0 \leq s < t \longrightarrow 0 \leq s < t_0$

p. 26, left line 3 and line 6 :
(1980) $\longrightarrow$ (1981), pp. 335 $\longrightarrow$ pp. 355

PREDICTION OF EARTHQUAKE-INDUCED DEFORMATION OF EARTH DAMS*

Closure by Eiichi Taniguchi**, Robert V. Whitman*** and W. Allen Marr****

The writers thank Bouckovalas for his insightful discussion.

Bouckovalas has used a particular form of stress-strain relation to calculate the difference in strain for triaxial and plane-strain conditions. This certainly is a potentially important point, which deserves further attention. However, the state-of-the-art in the calculation of permanent strains is such that a possible 33% error is, today, small compared to other uncertainties in any analysis.

Bouckovalas also argues that adding the seismic force rightwards, then leftwards, and superimposing the results is not necessary since the cyclic stress-strain relation already includes stressing in both directions. This argument overlooks a problem in the analysis as we performed it. We use a stress-strain relation between static plus cyclic force and

* Vol. 23, No. 4, December 1983, pp. 126-132. (Previous discussion by G. Bouckovalas, Vol. 24, No. 2, pp. 120-121.)
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